Investigation of Geometric Variations for Multicell Cavities Using Perturbative Methods

Korinna Brackebusch, Ursula van Rienen, *Member, IEEE*

Institute of General Electrical Engineering, University of Rostock, Albert-Einstein-Straße 2, 18059 Rostock, Germany

The design of multicell accelerator cavities is a challenging task since it implies the manipulation of various shape parameters regarding different (partially contradicting) optimization goals. Simulating the electromagnetic characteristics of the full structure depending on various geometric parameters typically involves an enormous computational effort. In most cases, this limits the observed frequency range and the number of optimization passes. For the same reason, the effects of unintended shape deviations are usually excluded from optimization processes, even though they may be of particular importance for the final design.

Perturbative methods offer an efficient approach to tackle this issue. They allow for the computation of the eigenmodes and the derived cavity performance parameters for a vast number of cavity designs based on one initial design. In this contribution, we investigate the applicability of perturbative methods for performance optimization and simultaneous consideration of shape variations of a multicell structure.

***Index Terms*—Electromagnetic Fields, Perturbative Methods, Numerical Simulation, Cavity Resonators, Design Optimization.**

I. Introduction

ONE of the main challenges in designing a superconducting multicell accelerating cavity is to provide the best possible beam acceleration and simultaneously guarantee an adequate damping of higher-order modes (HOMs). This requires an eigenmode analysis of the closed RF structure over a large frequency range. The standard approach is the use of a 3D eigenmode solver for an iterative modification of the structure's shape to obtain the best design parameters. The drawback of this approach is that each modification requires a full simulation, regardless of its extent. The resulting computational effort is enormous, forcing a tradeoff between design requirements and computational time.

Perturbative methods are based on the idea that a moderate shape modification only affects the eigenmodes to a likewise moderate degree. Inefficient repetitive computations are avoided by deriving the (so-called perturbed) eigenmodes of a modified shape directly from the (so-called unperturbed) eigenmodes of an initially chosen shape. Solely the unperturbed modes have to be computed in a conventional way and this only once. Perturbative methods are particularly suitable to improve the efficiency of investigations of a multitude of geometric variations. A further substantial advantage is the fact that the modes of a structure with symmetry-breaking elements (e.g. tuned end cells) can be derived from the modes of a fully symmetric structure. For this purpose, it is only necessary to compute the unperturbed modes for different symmetry settings and to merge them. The methods were successfully demonstrated for single cell cavities [1]. They are now applied to multicell cavities by exemplarily studying the main linac multicell cavity of the Berlin Energy Recovery Linac Project (bERLinPro) [2]. The 1.3 GHz elliptical 7-cell cavity is based on a modified Cornell ERL design. Eight different preliminary candidates for the final design (state: middle of 2014) are compared and the effects of unintended shape deformations, that may be caused by manufacturing tolerances, are investigated by a distinct variation of the equator and iris radii of the individual cells.

II. Theory and Realization

The method presented here is a special generalization of Slater's theorem (GST) [3] applicable to cavities with perfect electric or perfect magnetic boundary conditions. A detailed description of the method is available in [4], [5], [1]. This paper solely recapitulates the most relevant facts. The perturbed modes of a modified shape are expanded in terms of the unperturbed ones by analyzing the interactions of the unperturbed modes inside the volume ΔV that is removed by the modification from the initial shape (see Fig. 1). The interaction terms (ITs) of the modes are composed of their frequencies f and the integrals of the scalar products of their electric ($\mathbf{E}$) and magnetic fields ($\mathbf{H}$), respectively over the removed volume ΔV (see (6) in [5]). The unperturbed modes are computed with the 3D Eigenmode Solver of CST MWS [6] and exported to Mathematica [7] that is used for the implementation of GST.

The initial (unperturbed) shape, shown in Fig. 1, is defined in such a way that its end cells include all eight bERLinPro design candidates. In addition, its equator and iris radii are 0.5 mm larger than the ones of the actual bERLinPro design to allow for a radial variation of ± 0.5 mm. The initial shape is fully symmetric so that all three symmetry planes can be used to speed up the simulations. For an efficient computation of the interaction terms the full structure is partitioned into twelve

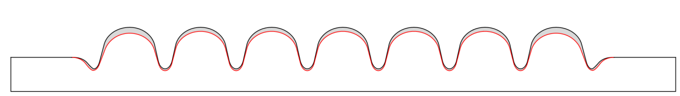

Fig. 1. Initial cavity shape (black) used for the computation of the unperturbed modes. The eigenmodes for any modified shape (red) can be derived from the interaction of the unperturbed modes inside the volume ΔV (gray).

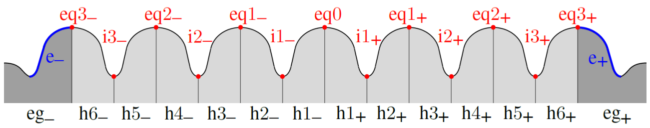

Fig. 2. Partitioning of the initial cavity shape into half cells (h) and end cell groups (eg) to allow for an individual variation of equator (eq) and iris (i) radii as well as end cell shapes (e).

half cells and two end cell groups as shown in Fig. 2. So, the ITs of each segment can be processed separately. To investigate radial variations of 0.5, 0 and -0.5 mm, eight different sets of ITs for each half cell and 24 for each end cell group have to be computed. Furthermore, only the ITs of six half cells ($h1_+$ to $h6_+$) in the positive longitudinal coordinate range have to be determined from the exported field data. The ITs in the negative range ($h1_-$ to $h6_-$) are directly deduced from the ones in the positive range based on the symmetry settings of the structure. Combining the ITs of the individual 14 segments to a full structure delivers the eigenmodes of about 12.7 million different designs.

III. Results

The frequencies of the eigenmodes can be computed with an adequate accuracy over a wide frequency range. The relative error is $5 \cdot 10^{-4}$ at maximum for frequencies up to 7.1 GHz (compared to reference simulations done with CST MWS). The majority of the modes has an error less than 10^{-4}. Likewise, the perturbed fields coincide very well as Fig. 3 demonstrates for the accelerating mode.

Exemplarily for the effects of shape variations, the frequency shifts that may arise from a possible deviation of the actual shape from the ideal one is shown for candidate 1 in Fig. 4. All half cells were equally modified and Table I lists the applied radii modifications. Increasing the equator radii reduces the frequencies while decreasing them results in the exact opposite. Thus, the magnitude of the frequency shifts mainly depends on the equator radii, less on the iris radii, as it is particularly apparent for the monopole modes in Fig. 4. In contrast, the iris radii influence the cell-to-cell coupling and thereby primarily affect the frequency spread within one passband. However, the spread also strongly depends on the electromagnetic field at

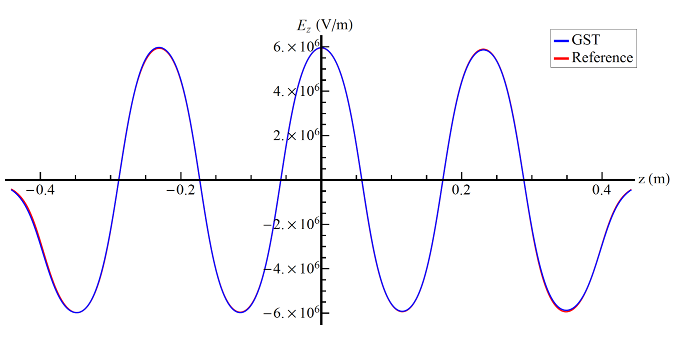

Fig. 3. Longitudinal electric field E_z of the 1.3 GHz π-mode along the beam axis z for candidate 1. The GST based field (blue) deviates less than $1.4 \cdot 10^5$ V/m (2.4% of the maximal value) from the reference field (red). The obtained field flatness is 98.07% (reference: 98.67%).

TABLE I
Settings for equator and iris radii variations.

Variation	1	2	3	4	5	6	7	8	9
eq. (mm)	0.5	0.5	0.5	0	0	0	-0.5	-0.5	-0.5
iris (mm)	0.5	0	-0.5	0.5	0	-0.5	0.5	0	-0.5

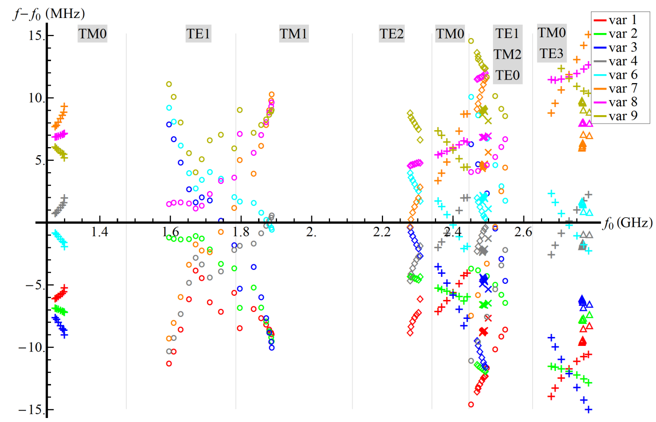

Fig. 4. Frequency shift for first ten passbands of candidate 1 caused by a variation of equator and iris radii. f_0 represents the respective frequency of the ideal shape (variation 5).

the irises. The comparison of the first TM_0 and the first TE_1 passband demonstrates this effect.

The full contribution will present the analysis of the modal spectrum up to 7.1 GHz including an investigation of performance parameters (e.g. accelerating voltage, shunt impedances, field flatness, peak field ratio) of the different candidates in due consideration of shape variations.

IV. Conclusion

Perturbative methods constitute an efficient approach for the eigenmode computation for a multitude of different cavity designs exceeding the limitations of conventional approaches. By a suitable choice and partitioning of one initial shape, they allow to optimize a design and to simultaneously investigate the effects of unintended shape deviations over a large frequency range with a moderate computational effort.

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